Autopilot Design for a Missile with Reaction-Jet Using Coefficient Diagram Method

Rui Hirokawa*, Koichi Sato†
*Mitsubishi Electric Co., Kamakura Works, Kanagawa, Japan
†Fujisawa, Kanagawa, Japan

Advanced guidance missiles employ blended control of aero-fin and reaction-jet to improve the guidance performance against highly maneuverable targets. The blended control requires multiple inputs and multiple outputs (MIMO) control system design, the design process is much complex than that of conventional aero-fin autopilot. This paper describes a autopilot design for a missile with aero-fin and reaction-jet using the Coefficient Diagram Method (CDM). Performance and stability trade-off study using the CDM is summarized. It is shown that MIMO autopilot design problem can be decomposed into feedback and feedforward controller design problems. A reasonable form of the controller is suggested using the coefficient diagram. A feedback controller is designed using the algebraical approach. The form of the controller has the same form as the conventional design. A wash-out type feedforward controller achieving the minimum fuel consumption is designed using the same manner. It is shown that the feedforward controller can be designed independently of the feedback controller. Improved response and robustness against the reaction-jet force saturation of the designed control system is proven by the simulation.

Nomenclature

\( \alpha \) angle of attack, rad
\( m \) mass, kg
\( I_{yy} \) inertia in pitch-axis, kg-m\(^2\)
\( \delta_a \) aero-fin angle, rad
\( \delta_s \) throttle of reaction jet
\( b \) arm length from e.g., m
\( V \) velocity, m/s
\( f_s \) force of reaction jet, N
\( q \) pitch rate, rad/s
\( a \) acceleration, m/s\(^2\)
\( a_c \) acceleration command, m/s\(^2\)
\( \gamma_i \) stability index
\( \tau \) equivalent time constant, s

Introduction

Requirement for the improvement of homing guidance performance against highly maneuverable targets in the future guided missiles will needs the control system having faster response. The reaction-jet control system (RCS) will be used to improve the response of the future guided missiles.

The high response can be reached with the RCS in low dynamic pressure condition, however the maximum control force and the maximum operation time of the RCS are constrained by missile configuration such as weight and size. The MIMO control system blending the conventional aero-fin control and reaction-jet control, can overcome such constraints and reach the high response and the high agility.

There are two types of RCS configuration, the divert type and the moment type. The RCS is located near the center of gravity (c.g.) arising the acceleration directly in former type, and is located in ahead of or in rear of c.g. arising the force and the moment in the pitch axis in later type. Interference between the rotational and divert motion of the divert type is far less than that of the moment type, so the controller of the divert type can easily designed with simple decoupling compensation.\(^1,2\)

Because the rotational moment is arisen by the aerodynamic control and the reaction-jet control in the moment type configuration, the rotational dynamics has severe dynamic coupling, the controller of the moment type should be designed as MIMO controller. However, the typical conventional missile controller is based on the SISO design method, so the new control design method is necessary.

In previous research, some papers dealing with the control design of missile with RCS. Weil et al. proposed the blended aero and reaction jet missile autopilot formulating as the variable structure systems and indicated the decreasing of the fin-rate could be reached by the blended control.\(^3\) Innocenti also pro-
posed the mixed aerodynamic and RCS controller for high angle of attack maneuver using the mixture of aerodynamic and reaction-jet control scheduling with the angle of attack. Wise et al. proposed the autopilot for aero-fin controlled missile with the RCS using linear quadratic regulator technique, however the detail of the controller design is not described in the paper. Menon et al. proposed adaptive techniques for multiple actuator blending using fuzzy control. Chadwick proposed the simple aerodynamic and lateral-thrust controller to improve the guidance performance of missiles against weaving targets in high altitude.

The purpose of this paper is to design MIMO autopilot balancing the stability and the response for a aero-controlled missile with reaction-jet. In this paper, Coefficient Diagram Method (CDM) is used to design the autopilot. In the CDM, "Coefficient Diagram" is fully utilized instead of Bode diagram or root locus plot. The coefficient diagram is a semi-log plotted diagram where the coefficients of characteristic polynomials are shown in the coordinate with log scale.

The stability of its plant is measured by the convexity of the curve. The response is measured by the inclination of the curve. The variation of the shape of the curves due to plant/controller parameter variation is a measure of robustness.

Successful applications were reported in various fields of controls, however all of these design are for SISO or SIMO systems. Recently, Manabe proposed the MIMO controller for a dual-control surface missile using the CDM composed by the feedback and the feedforward controller.

In this paper, the controller is decomposed to the feedback control by aerodynamic control and the feedforward control by reaction-jet control. These controllers are designed by the CDM.

Design complexity such as the trade-off between the performance and the stability in the MIMO design is a major problem. The multi-step design approach using CDM for the feedback and the feedforward controller design is proposed in this paper to overcome the difficulty.

This paper is composed as follow, At first, the design basics of the CDM are summarized. Next, the mathematical model of a missile with RCS is presented. After this, the basic control structure is determined using CDM and a feedback controller designed as a SIMO problem. A series of feedforward controllers are suggested in the next section. At last, simulation results are shown for these controllers.

**Coefficient Diagram Method**

**Basics**

The CDM is an algebraic control design approach with the following features. Polynomial representation is not only accurate as state space, but also easy to understand as transfer function. Simultaneous design of characteristic polynomial and controller gives a well-balanced and practical controller, whereby strong robustness is usually guaranteed.

The CDM design is based on stability indices and equivalent time constant as defined later. Thus, a controller of the lowest order with the narrowest bandwidth and no-overshoot can be easily designed for the specified settling time. The CDM can be considered as a class of generalized conventional PID method, and more reliable parameter selection rules are provided.

**Mathematical Model**

Standard block diagram of the CDM design is shown in Figure 1. This diagram is valid for SISO, SIMO, and MIMO systems.

In general, variables and components are defined by vectors and matrices of proper dimensions respectively. The plant equations are given as,

\[ A_p(s)x = u + d \]  
\[ y = B_p(s)x \]

where \( x, u, y \) and \( d \) are state, input, output, and disturbance variables. \( A_p(s) \) and \( B_p(s) \) are the denominator and numerator polynomial matrix of the plant respectively. In addition, it will be easily seen that these expressions have a direct correspondence with the control canonical form of the state-space expression, and \( x \) corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of \( x \) of higher order. The plant transfer function \( G_p(s) \) is given in the right co-prime fraction form.

\[ G_p(s) = B_p(s)A_p(s)^{-1} \]
Controller equation is given as,

$$A_c(s)u = B_a(s)y_r - B_c(s)(y + n)$$  \hspace{1cm} (3)

where $y_r$ and $n$ are reference inputs and plant noises. $A_c(s)$ is the denominator polynomial matrix of the controller. $B_a(s)$ is the reference polynomial matrix, and $B_c(s)$ is the feedback numerator polynomial matrix of the controller. As the controller transfer function has two numerators, it is a two-degree-of-freedom control system. This expression corresponds to the observer canonical form of the state-space expression. The reference and feedback transfer function of the controller $G_a(s)$ and $G_p(s)$ is expressed in the left co-prime fraction form.

$$G_a(s) = A_c(s)^{-1}B_a(s)$$  \hspace{1cm} (4a)

$$G_c(s) = A_c(s)^{-1}B_c(s)$$  \hspace{1cm} (4b)

Elimination of $y$ and $u$ from Eq.(3) by Eqs.(1) gives,

$$A(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n$$  \hspace{1cm} (5)

where $A(s)$ is the closed-loop system polynomial matrix and given as,

$$A(s) = A_c(s)A_p(s) + B_c(s)B_p(s)$$  \hspace{1cm} (6)

The characteristic polynomial $P(s)$ is given as,

$$P(s) = \det A(s)$$  \hspace{1cm} (7)

In the CDM design, MIMO problem is decomposed into a series of SIMO problem. In SIMO problem, $A(s)$ is replaced by $P(s)$, and there are 9 transfer functions corresponding to 3 inputs and 3 outputs. Because only 3 components, $A_c(s)$, $B_c(s)$ and $B_a(s)$ are to be designed, only 3 transfer functions are required for design. In the CDM, the following four basic relations are selected as standard,

$$P(s)x = P(0)y_r$$  \hspace{1cm} (8a)

$$P(s)y = B_p(s)B_a(s)y_r$$  \hspace{1cm} (8b)

$$P(s)y = B_p(s)A_c(s)d$$  \hspace{1cm} (8c)

$$P(s)(-u) = B_c(s)B_p(s)d$$  \hspace{1cm} (8d)

Eq.(8a) is the response of $x$ to $y_r$ when $B_a(s) = P(0)$, and it specifies the characteristic polynomial. Eq.(8b) is for the command following characteristics. Eq.(8c) is for the disturbance rejection characteristics. Eq.(8d) corresponds to the complementary sensitivity function $T(s)$, and it is useful to check the robustness. In the CDM design, the four basic relations are used as performance specification. The design of $P(s)$ is initially made to satisfy specifications on Eqs. (8a), (8c), (8d), and then $B_a(s)$ is adjusted to satisfy the specification on Eq.(8b).

### Basic Relations

Some mathematical relations extensively used in CDM will be introduced hereafter. These relations will be used in later section. The characteristic polynomial $P(s)$ is given as the following form.

$$P(s) = a_ns^n + \ldots + a_1s + a_0 = \sum_{i=0}^{n} a_is^i$$  \hspace{1cm} (9)

Stability index $\gamma_i$, equivalent time constant $\tau$, and stability limit $\gamma_i^*$ are defined as follow.

$$\gamma_i = a_i^2/(a_i - a_{i+1}), \hspace{1cm} i = 1 \ldots n - 1$$ \hspace{1cm} (10a)

$$\tau = a_1/a_0$$ \hspace{1cm} (10b)

$$\gamma^*_i = 1/\gamma_i - 1/\gamma_{i+1}, \hspace{1cm} i = 1 \ldots n - 1$$ \hspace{1cm} (10c)

where $\gamma_0$ and $\gamma_n$ are defined as $\infty$.

The equivalent time constant of the $i$-th order $\tau_i$ is defined in the similar manner as $\tau$ as follow,

$$\tau_i = a_{i+1}/a_i$$ \hspace{1cm} (11)

By Eqs.(10a) and (11),

$$\tau_i/\tau_{i-1} = (a_{i+1}/a_i)/(a_i/a_{i-1}) = 1/\gamma_i$$ \hspace{1cm} (12a)

$$\tau_i = \tau_i/\tau_1 a_0$$ \hspace{1cm} (12b)

By using Eqs.(11) and (12b) repeatedly, $a_i$ is expressed by $\tau_i$ and $a_0$.

$$a_i = \tau_{i-1} \tau_{i-2} \ldots \tau_1 \tau a_0$$

$$= a_0 \tau_i \prod_{k=1}^{i-1} \gamma_i^{k-1}, \hspace{1cm} i \geq 2$$ \hspace{1cm} (13)

Then characteristic polynomial is expressed by $a_0, \tau$ and $\gamma_i$ as follow,

$$P(s) = a_0 \sum_{i=2}^{n} \left( \tau s \prod_{k=1}^{i-1} \gamma_i^{k-1} \right) + \tau s + 1$$ \hspace{1cm} (14)

Sufficient condition for stability and instability by Lyapunov constitutes the theoretical basis of the CDM. It states as follow. “The system of any order is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd polynomial is unstable.” Thus the sufficient condition for stability is given as,

$$a_i > 1.12 \left[ \frac{a_{i-1}}{a_{i+1}} + \frac{a_{i+1}}{a_{i-1}} \right]$$

$$\gamma_i > 1.12 \gamma^*_i$$  \hspace{1cm} (15)

The sufficient condition for instability is given as,

$$a_{i+1}a_i \leq a_{i+2}a_{i-1}, \hspace{1cm} \gamma_{i+1} \gamma_i \leq 1$$  \hspace{1cm} (16)
As these conditions are graphically expressed in the coefficient diagram, designers can intuitively assess the stability of the system. Fig. 2(a) is a 4th-order example. Point A is obtained by drawing a line from a in parallel with line a_3. Similarly, point B is obtained by drawing a line from a_0 in parallel with line a_3 a_1. The stability condition is a_2 > (A + B). The outer condition is \( \gamma_2 > \gamma_1^2 \). Fig. 2(b) is a 3rd-order example. Point A is \( a_3 \) and point B is \( a_0 \). Thus if A is below B, the system is unstable. Point C is \( \sqrt{\gamma_2 \gamma_1} \). If it locates below 1, the system is unstable.

In the CDM, following stability indices are recommended:

\[
\gamma_2 = \gamma_3 = \ldots = \gamma_{n-1} = 2, \quad \gamma_1 = 2.5
\]

For more relaxed form, with very small sacrifice of stability,

\[
\gamma_i > 1.5 \gamma_i^*, \quad i = n - 1 \ldots 4
\]

\[
\gamma_2 = \gamma_3 = 2, \quad \gamma_1 = 2.5
\]

In these cases, the step response of Eq. (7) has no overshoot, and the settling time is about 2.5...3τ.

**Design Procedure of The CDM**

Design steps taken at SISO or MIMO design in the CDM are as follow.

1. Assume the structure of a controller, such as the order of controller, and low or and high frequency characteristics.
2. Assume the possible range of equivalent time constant τ, where a rough sketch of the coefficient diagram is very helpful.
3. Assume the stability index in the standard form.
4. Find the characteristic polynomial by Eq. (14).
5. Solve the Diophantine equation of Eqs. (6) (7).
6. In some difficult case of the problems, the design may not be satisfactory. In such cases, τ and \( \gamma_i \) must be adjusted. The coefficient diagram and the sufficient condition by Lyapatov\textsuperscript{13, 14} are very helpful at this stage.

**Model Definition of a Missile**

An aero-fin controlled missile with the RCS is shown in Figure 3. This example is picked up from an open literature.\textsuperscript{7} The RCS is located in front of c.g., the distance between c.g. and the RCS location is b. The design condition is assumed to be 12.6km in altitude and 1219m/s in velocity. The required maximum normal acceleration is 200m/s\(^2\). The fin actuators are represented by time lag of 0.02s, and the RCS dynamics is represented by time lag of 0.002s. The aero-fin and the RCS parameters of an ideal mid-range missile are defined in Table 1.

<table>
<thead>
<tr>
<th>V</th>
<th>altitude</th>
<th>Z_a</th>
<th>M_a</th>
<th>Z_{\delta a}</th>
<th>M_{\delta a}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1219m/s</td>
<td>12.6km</td>
<td>-0.4605</td>
<td>-41.858</td>
<td>0.0276</td>
<td>26.7891</td>
</tr>
<tr>
<td>14.546</td>
<td></td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26.7891</td>
<td></td>
<td>0.0276</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With the assumption of the linear thrust, \( f_s \), the force by RCS is defined using \( \delta_s \) as follow,

\[
f_s = f_{s\text{max}} \delta_s
\]

The dimensional coefficients are defined as follow,

\[
M_{\delta a} = \frac{f_{s\text{max}} b}{I_{yy}}, \quad Z_{\delta a} = \frac{f_{s\text{max}}}{mV}
\]

where \( f_{s\text{max}} \) is the maximum force by the RCS.

The state is \( x = [a, q, \alpha] \), control input is \( u = [\delta_s, \delta_a] \), output is \( y = [a, q, \alpha] \). The linear system equations of rotational motion in a missile body frame are defined as follow,

\[
\begin{align*}
\frac{dx}{dt} &= A_m x + B_m u \quad (21a) \\
y &= C_m x + D_m u \quad (21b)
\end{align*}
\]

where,

\[
A_m = \begin{bmatrix} Z_a & 1 \\ M_a & 0 \end{bmatrix}, \quad B_m = \begin{bmatrix} Z_{\delta a} & Z_{\delta a} \end{bmatrix}
\]

\[
C_m = \begin{bmatrix} -V Z_a & 0 \\ 0 & 1 \end{bmatrix}, \quad D_m = \begin{bmatrix} -V Z_{\delta a} & -V Z_{\delta a} \\ 0 & 0 \end{bmatrix}
\]
From Eqs.(1a),(21) and Table 1,

\[ A_p(s) = B_m^{-1}(sI - A_m) = \begin{bmatrix} a_{p11} & a_{p12} \\ a_{p21} & a_{p22} \end{bmatrix} \]  

\[ B_p(s) = C_m + D_m A_p = \begin{bmatrix} -1219s & 1219 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \]  

\[ a_{p11} = -18.18s - 7.587, \ a_{p12} = 0.0187s + 18.18 \]

\[ a_{p21} = 9.870s + 5.682, \ a_{p22} = 0.0271s - 9.870 \]

### Controller Design

#### Control Structure Design

In order to analyze the basic control structure, the input-output relations are derived from Eq.(22) as follow,

\[ \begin{bmatrix} a \\ q \end{bmatrix} = \frac{1}{A_{pd}(s)} \begin{bmatrix} b_{as} & b_{aa} \\ b_{qs} & b_{qa} \end{bmatrix} \begin{bmatrix} \delta_s \\ \delta_a \end{bmatrix} \]  

where,

\[ A_{pd}(s) = \det B_m(s) \det A_p(s) = s^2 + 0.4605s + 41.8580 \]  

and,

\[ b_{as} = 48.78s^2 + 10208, \ b_{aa} = -33.69s^2 + 13630 \]

\[ b_{qs} = 14.55s + 8.373, \ b_{qa} = 26.79s + 11.18 \]

\[ b_{as} = -0.040s + 14.55, \ b_{aa} = 0.0276s + 26.789 \]

These polynomials are shown in the coefficient diagram of Figure 4, where the coefficients of \( s^i \) are multiplied by \( 10^i \) for convenience. The minus value is indicated by (―).

First order coefficient of the denominator \( A_{pd}(s) \) is too small, therefore it should be compensated by feedback control.

This coefficient can be modified by proper feedback involving \( b_{qa} \) and \( b_{qs} \). Because these two signals are almost same, either of them can be used. The 0-th order coefficient can be modified by \( b_{as} \), \( b_{aa} \) and \( b_{as} \). However \( b_{aa} \) can not be used, because the fuel consumption of the RCS should be minimized.

From this analysis, the most reasonable feedback control is to control \( \delta_a \) by \( a \) and \( q \), while keeping \( \delta_s \) equals to zero.

The signals \( a \) and \( \alpha \) are similar in nature. Although \( a \) feedback is necessary for the acceleration control, \( \alpha \) control is not necessary.

\( \alpha \) can be controlled effectively by feedforward control. \( \alpha \) control can be used to improving the guidance performance by reducing the sensitivity of radome parasitic effects.

The proportional feedback of \( a \) is not undesirable, because the second order coefficient of \( b_{aa} \) is negative and will degrade the robustness of the system.

#### Feedback Controller Design

By defining \( \delta_a = 0 \), the plant becomes a SIMO system and expressed as follow,

\[ \begin{bmatrix} a \\ q \end{bmatrix} = \frac{1}{A_{pd}(s)} \begin{bmatrix} b_{aa} \\ b_{qa} \end{bmatrix} \begin{bmatrix} \delta_a \end{bmatrix} \]  

The PI controller with the feedback \( q \) and \( a \) is defined as,

\[ su_r = k_0 a_r - [(k_2 s + k_1) q + k_0 a] \]  

This controller has the same structure as the conventional Raytheon 3-loop autopilot design. The block diagram is shown in Figure 5. From Eqs.(6),(7), the characteristic polynomial \( P(s) \) becomes,

\[ P(s) = s A_{pd}(s) + (k_2 s + k_1) b_{qa} + k_0 b_{aa} = s^3 + a_2 s^2 + a_1 s + a_0 \]  

This is a 3rd order system, and \( \gamma_1 \) and \( \gamma_2 \) are chosen as the standard value (\( \gamma_1 = 2.5, \ \gamma_2 = 2 \)). Because the neglected actuator dynamics has time constant of 0.02s, the time constant of the highest order \( \gamma_2 \) should be twice larger than 0.02s at least for the stability requirement. \( \gamma_2 \) is selected as 0.04s from this constraint, \( \tau \) is defined as 0.2s from Eq.(12b).

Then by Eq.(13),

\[ a_2 = 25, \ a_1 = 312.5, \ a_0 = 1562.5 \]
When only $a$ is considered as output, the relations become,

$$ a = \frac{1}{P(s)} \left( b_{11} \delta_s + b_{12} a^*_r \right) $$  \hspace{1cm} (32)$$

where $a^*_r$ is the acceleration reference of the feedback controller, designed in the previous section.

To uncouple the feedforward controller design from the feedback control system, $a^*_r$ and $\delta_s$ are redefined using the decoupling gain $k_3,k_4$.

$$ \begin{bmatrix} \delta_s \\ a^*_r \end{bmatrix} = \begin{bmatrix} k_4 & 0 \\ k_3s+1 & 1 \end{bmatrix} \begin{bmatrix} a_f \\ a_r \end{bmatrix} \hspace{1cm} (33)$$

Eq.(32) becomes

$$ a = \frac{1}{P(s)} \left( b_{11}^* a_f + b_{12} a_r \right) $$  \hspace{1cm} (34)$$

where,

$$ b_{11}^* = k_4 b_{11} + (k_3 s + 1) b_{12} $$  \hspace{1cm} (35)$$

If the approximation, shown in below, is composed,

$$ P(s) \approx b_{11}^* $$  \hspace{1cm} (36)$$

the acceleration response is represented by the simple linear relationship as below,

$$ a = a_f + \frac{|b_{12}|}{P(s)} a_r $$  \hspace{1cm} (37)$$

By comparison of the coefficients on the both sides in Eq.(36), the parameters are derived as follow,

$$ k_4 = 0.0153, \ k_3 = -0.0699 \hspace{1cm} (38)$$

From Eqs.(35),

$$ b_{11}^* = 1.016 s^3 + 25.00 s^2 + 312.5 s + 1562.5 \approx P(s) $$  \hspace{1cm} (39)$$

With the relation of Eq.(39), the feedforward controller could be designed with the feedback system independently.

A feedforward controller is defined as follow,

$$ G_{af}(s) = \frac{k_5 s}{l_2 s^2 + l_1 s + 1} $$  \hspace{1cm} (40)$$

The wash-out type is selected because the small steady gain of the reaction-jet is preferred. Using the co-prime factored representation with the decoupling parameters,

$$ A_f(s) \begin{bmatrix} \delta_s \\ a^*_r \end{bmatrix} = B_f(s) a_r $$  \hspace{1cm} (41a)$$

$$ A_f(s) = \begin{bmatrix} l_2 s^2 + l_1 s + 1 \\ -(k_3 s + 1)/k_4 \end{bmatrix} $$  \hspace{1cm} (41b)$$

$$ B_f(s) = \begin{bmatrix} k_3 k_5 s \\ 1 \end{bmatrix} $$  \hspace{1cm} (41c)$$

The value of $m_0 = 0.1146$ is obtained from the steady state gain.

The coefficient diagram is shown in Figure 6.

Feedforward Controller Design

The closed-loop polynomial matrix equation can be obtained from Eqs.(22),(26),

$$ A(s) = A_c(s) A_p(s) + B_c(s) B_p(s) $$  \hspace{1cm} (30a)$$

$$ A_c(s) = \begin{bmatrix} 1 & 0 \\ 0 & s \end{bmatrix}, \ B_c(s) = \begin{bmatrix} 0 & 0 & 0 \\ k_0 & k_2 s + k_1 & 0 \end{bmatrix}, $$

$$ B_a(s) = \begin{bmatrix} 1 & 0 \\ 0 & m_0 \end{bmatrix} $$  \hspace{1cm} (30b)$$

From Eqs (22) and (24), the input-output relations are obtained as,

$$ \begin{bmatrix} a \\ q \end{bmatrix} = \frac{1}{ \det A(s)} B(s) \begin{bmatrix} \delta_s \\ a_r \end{bmatrix} $$  \hspace{1cm} (31a)$$

$$ B(s) = B_p(s) \text{adj} A(s) B_a(s) $$

$$ \equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} $$  \hspace{1cm} (31b)$$

where,

$$ \det A(s) = P(s) = s^3 + 25 s^2 + 312.5 s + 1562.5 $$

$$ b_{11} = 48.7747 s^3 + 1886.8 s^2 + 27572 s $$

$$ b_{12} = -3.8614 s^2 + 1562.5 $$

Fig. 5 Feedback Control System

Fig. 6 Coefficient diagram of feedback system
The block diagram of the control system with feedforward controller in co-prime factorization form is shown in Figure 7. The controller is represented by a serial connection of the feedback and feedforward controllers. The block diagram in normal form is also shown in Figure 8 where control decoupling compensator \( M_{dc} \) defined as follows,

\[
M_{dc}(s) = \begin{bmatrix} k_4 \\ k_3 s + 1 \end{bmatrix}
\]  

(42)

The controller is represented by the connected form of the conventional 3-loop feedback controller, the decoupling path and the feedforward controller.

Here, the gain parameters \( k_3, l_1, k_5 \) of the feedforward controller are defined from the required response of \( a_r \). These parameters can be selected independently of the feedback system.

Considering the faster response and the minimum overshoot, \( k_5 = 0.08, l_1 = 0.112, l_2 = 0.0056 \) were selected. The coefficient diagram is shown in Figure 9. The coefficients of the denominator and the numerator are shown in this figure. The denominator is represented by the solid line, the numerator with and without the feedforward controller are represented by the dashed line. The numerator with feedforward control forms the convex form with the larger positive curvature than that of the denominator. On the other hand, the numerator without feedforward has some negative coefficients and the negative curvature, this form is caused by the non-minimum phase zero of the plant.

**Simulation**

Some simulations were performed with the designed controller Eq.(41), where the exact missile model in Table 1, including actuator dynamics, is used.

Figure 10 is the simulation result of the feedback control system by aero-fin control. The commanded acceleration is 5G, the maximum acceleration of the RCS is assumed to be 5G. The initial negative response of acceleration is caused by the non-minimum phase zero of the plant.

The results of the control system with feedforward control is shown in Figure 11. The acceleration caused by the reaction jet is described by the dashed line in the left-upper figure. The negative aero-fin angle \( \delta_a \) is used in accordance with the feedforward control by the RCS. The response of the acceleration for the both system is compared in Figure 12. The rising time of the response is improved considerably by the feedforward control.

Another simulation with against higher acceleration command, 20G, was also performed to assess the robustness of the control system under the saturation constraints of the RCS force. The result is shown in Figure 13. Because the commanded acceleration is higher than the acceleration limit of the RCS, the output of the feedforward controller was saturated in the initial response. With considering the RCS limit, the control system is still maintaining the stability.
Concluding Remarks

This paper described the autopilot design for a missile with aerodynamic fin and reaction jet using CDM. The results of the trade-off study is summarized. It is shown that the MIMO autopilot design problem can be decomposed into the feedback and feedforward controller design problems. Improved response and the robustness against the RCS force limit of the designed control system are proven by the simulation.

References